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INTERNATIONAL UNION FOR THE PROTECTION OF NEW VARIETIES OF PLANTS

GENEVA

Associated Document
to
The General Introduction to the Examination
of Distinctness, Uniformity and Stability and the
Development of Harmonized Descriptions of New Varieties of Plants (document TG/1/3)

DOCUMENT TGP/10

"EXAMINING UNIFORMITY"

Section TGP/10.3.2: Statistical Methods: Offtypes

Document prepared by experts from the United Kingdom, Denmark and Germany

to be considered by the

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SECTION 10.3.2

STATISTICAL METHODS: OFFTYPES

TESTING UNIFORMITY BY OFF-TYPES – FIXED POPULATION STANDARD

SUMMARY

1. This section describes the method of assessing uniformity by comparing the number of off-types observed to a fixed population standard. This is of particular use for self-pollinated and vegetatively propagated crops.

2. Methods for assessing uniformity using off-types for other types of crop are in development.

3. The maximum number of off-types that is acceptable should be chosen so that the probability of rejecting a candidate variety that should meet the crop standard is low. On the other hand the probability of accepting a candidate variety that has many more off-types than the standard of that crop should also be low.

4. The methods described here address the problem of choosing the maximum permitted number of off-types for different standards and sample sizes so that the probability of making errors is known and acceptable. The methods involve establishing a standard for the crop in question and then choosing the sample size and the number of off-types that best satisfy the risks that can be tolerated.

5. This document also outlines procedures for when more than a single test (more than one year for instance) is done and mentions the possibility of using sequential tests to minimize testing effort.

INTRODUCTION

6. When testing for uniformity on the basis of a sample, there will always be some risk of making a wrong decision. The risks can be reduced by increasing the sample size but at a greater cost. The aim of the statistical procedure described here is to achieve an acceptable balance between risks.

7. The procedures described below require the user to define an acceptance standard (called the *population standard*) for the crop in question. The methods described then show how to determine the sample size and the maximum number of off-types allowed for various levels of risks.

8. The population standard is the maximum percentage of off-types that would be accepted if all individuals of the variety could be examined.

UPOV RECOMMENDATIONS ON THE FIXED POPULATION STANDARD METHOD OF ASSESSING UNIFORMITY BY NUMBER OF OFF-TYPES

9. This method is recommended for use in assessing the uniformity by number of off-types with a fixed population standard.

10. The sample size and acceptable number of off-types employed depend on the crop. Recommended sample sizes and acceptable numbers of off-types for different crops are given in the Annex to TGP/10.3.

ERRORS IN TESTING FOR OFF-TYPES

11. As mentioned, there will be some risk of making wrong decisions. Two types of error exist:

(a) Declaring that the variety lacks uniformity when it in fact meets the standard for the crop. This is known as "type I error."

(b) Declaring that the variety is uniform when it in fact does not meet the standard for the crop. This is known as "type II error."

12. The types of error can be summarized in the following table:

True state of the variety	Decision made on variety	
	Acceptance as uniform	Rejection as non-uniform
uniform	correctly accepted	type I error
heterogeneous	type II error	correctly rejected

13. The probability of correctly accepting a uniform variety is called the acceptance probability and is linked to the probability of type I error by the relation:

"Acceptance probability" + "probability of type I error" = 100%

14. The probability of type II error depends on "how heterogeneous" the candidate variety is. If it is much more heterogeneous than the population standard then the probability of type II error will be small and we will have a small probability of accepting such a variety. If, on the other hand, the candidate variety is only slightly more heterogeneous than the standard, we will have a large probability of type II error. The probability of acceptance will approach the acceptance probability for a variety with a level of uniformity near to the population standard.

15. Because the probability of type II error is not fixed but depends on "how heterogeneous" the candidate variety is, this probability can be calculated for different degrees of heterogeneity. This document gives probabilities of type II error for three degrees of heterogeneity: 2, 5 and 10 times the population standard.

16. In general, the probability of making errors will be decreased by increasing the sample size and increased by decreasing the sample size.

17. For a given sample size, the balance between the probabilities of making type I and type II errors may be altered by changing the number of off-types allowed.

18. If the number of off-types allowed is increased, the probability of type I error is decreased but the probability of type II error is increased. On the other hand, if the number of off-types allowed is decreased, the probability of type I errors is increased while the probability of type II errors is decreased.

19. By allowing a very high number of off-types it will be possible to make the probability of type I errors very low (or almost zero). However, the probability of making type II errors will now become (unacceptably) high. If only a very low number of off-types are allowed, the result will be a small probability of type II errors and an (unacceptably) high probability of type I errors. This will be illustrated by examples.

EXAMPLES

Example 1

20. From experience, a reasonable standard for the crop in question is found to be 1%. So the population standard is 1%. Assume also that a single test with a maximum of 60 plants is done. From tables 4, 10 and 16 (chosen to give a range of target acceptance probabilities), the following schemes are found:

Scheme	Sample size	Target acceptance probability*	Maximum number of off-types
a	60	90%	2
b	53	90%	1
c	60	95%	2
d	60	99%	3

21. From the figures 4, 10 and 16, the following probabilities are obtained for the type I error and type II error for different percentages of off-types (denoted by P_2, P_5 and P_{10} for 2, 5 and 10 times the population standard).

* See paragraph 54

Scheme	Sample size	Maximum number of off-types	Probabilities of error (%)			
			Type I	Type II		
				$P_2 = 2\%$	$P_5 = 5\%$	$P_{10} = 10\%$
a	60	2	2	88	42	5
b	53	1	10	71	25	3
c	60	2	2	88	42	5
d	60	3	0.3	97	65	14

22. The table lists four different schemes and they should be examined to see if one of them is appropriate to use. (Schemes a and c are identical since there is no scheme for a sample size of 60 with a probability of type I error between 5 and 10%). If it is decided to ensure that the probability of a type I error should be very small (scheme d) then the probability of the type II error becomes very large (97, 65 and 14%) for a variety with 2, 5 and 10% of off-types, respectively. The best balance between the probabilities of making the two types of error seems to be obtained by allowing one off-type in a sample of 53 plants (scheme b).

<u>Example 2</u>

23. In this example, a crop is considered where the population standard is set to 2% and the number of plants available for examination is only 6.

24. Using the tables and the figures 3, 9 and 15, the following schemes a-d are found:

Scheme	Sample size	Acceptance probability	Maximum number of off-types	Probability of error (%)			
				Type I	Type II		
					$P_2 = 4\%$	$P_5 = 10\%$	$P_{10} = 20\%$
a	6	90	1	0.6	98	89	66
b	5	90	0	10	82	59	33
c	6	95	1	0.6	98	89	66
d	6	99	1	0.6	98	89	66
e	6		0	11	78	53	26

25. Scheme e of the table is found by applying the formulas (1) and (2) shown in paragraph 46(f) of this document.

26. This example illustrates the difficulties encountered when the sample size is very low. The probability of erroneously accepting a heterogeneous variety (a type II error) is large for all the possible situations. Even when all five plants must be uniform for a variety to be accepted (scheme b), the probability of accepting a variety with 20% of off-types is still 33%.

27. It should be noted that a scheme where all six plants must be uniform (scheme e) gives slightly smaller probabilities of type II errors, but now the probability of the type I error has increased to 11%.

28. However, scheme e may be considered the best option when only six plants are available in a single test for a crop where the population standard has been set to 2%.

Example 3

29. In this example we reconsider the situation in example 1 but assume that data are available for two years. So the population standard is 1% and the sample size is 120 plants (60 plants in each of two years).

30. The following schemes and probabilities are obtained from the tables and figures 4, 10 and 16:

Sche-me	Sample size	Acceptance probability	Maximum number of off-types	Probability of error (%)			
				Type I	Type II		
					$P_2 = 2\%$	$P_5 = 5\%$	$P_{10} = 10\%$
a	120	90	3	3	78	15	<0.1
b	110	90	2	10	62	8	<0.1
c	120	95	3	3	78	15	<0.1
d	120	99	4	0.7	91	28	1

31. Here the best balance between the probabilities of making the two types of error is obtained by scheme c, i.e. to accept after two years a total of three off-types among the 120 plants examined.

32. Alternatively a two-stage testing procedure may be set up. Such a procedure can be found for this case by using formulae (3) and (4) later in paragraph 50.

33. The following schemes can be obtained:

Scheme	Sample size	Acceptance probability	Largest number for acceptance after year 1	Largest number before reject in year 1	Largest number to accept after 2 years
e	60	90	can never accept	2	3
f	60	95	can never accept	2	3
g	60	99	can never accept	3	4
h	58	90	1	2	2

34. Using the formulas (3), (4) and (5) (see paragraph 50) the following probabilities of errors are obtained:

Scheme	Probability of error (%)				Probability of testing in a second year
	Type I	Type II			
		$P_2 = 2\%$	$P_5 = 5\%$	$P_{10} = 10\%$	
e	4	75	13	0.1	100
f	4	75	13	0.1	100
g	1	90	27	0.5	100
h	10	62	9	0.3	36

35. Schemes e and f (which are identical) result in a probability of 4% for rejecting a uniform variety (type I error) and a probability of 13% for accepting a variety with 5% off-types (type II error). The decision is:

- Never accept the variety after 1 year
- More than 2 off-types in year 1: reject the variety and stop testing
- Between and including 0 and 2 off types in year 1: do a second year test
- At most 3 off-types after 2 years: accept the variety
- More than 3 off-types after 2 years: reject the variety

36. Alternatively, scheme h may be chosen but scheme g seems to have a too large probability of type II errors compared with the probability of type I error.

37. Scheme h has the advantage of often allowing a final decision to be taken after the first test (year) but, as a consequence, there is a higher probability of a type I error.

Example 4

38. In this example, we assume that the population standard is 3% and that we have 8 plants available in each of two years.

39. From the tables and figures 2, 8 and 14, we have:

Scheme	Sample size	Acceptance probability	Maximum number of off-types	Probability of error (%)			
				Type I	Type II		
					$P_2 = 6\%$	$P_5 = 15\%$	$P_{10} = 30\%$
a	16	90	1	8	78	28	3
b	16	95	2	1	93	56	10
c	16	99	3	0.1	99	79	25

40. Here the best balance between the probabilities of making the two types of error is obtained by scheme a.

INTRODUCTION TO THE TABLES AND FIGURES

41. In the TABLES AND FIGURES section, there are 21 table and figure pairs corresponding to different combinations of population standard and acceptance probability. These are design to be applied to a single off-type test. An overview of the tables and the figures are given in table A.

42. Each table shows the maximum numbers of off-types (k) with the corresponding ranges in sample sizes (n) for the given population standard and acceptance probability. For example, in table 1 (population standard 5%, acceptance probability ≥ 90%), for a maximum set at 2 off-types, the corresponding sample size (n) is in the range from 11 to 22. Likewise, if the maximum number of off-types (k) is 10, the corresponding sample size (n) to be used should be in the range 126 to 141.

43. For small sample sizes, the same information is shown graphically in the corresponding figures (figures (1 to 21). These show the actual risk of rejecting a uniform variety and the probability of accepting a variety with a true proportion of off-types 2 times (2P), 5 times (5P) and 10 times (10P) greater than the population standard. (To ease the reading of the figure, lines connect the risks for the individual sample sizes, although the probability can only be calculated for each individual sample size).

Table A. Overview of table and figure 1 to 18.

Population standard %	Acceptance probability %	See table and figure no.
10	>90	19
10	>95	20
10	>99	21
5	>90	1
5	>95	7
5	>99	13
3	>90	2
3	>95	8
3	>99	14
2	>90	3
2	>95	9
2	>99	15
1	>90	4
1	>95	10
1	>99	16
0.5	>90	5
0.5	>95	11
0.5	>99	17
0.1	>90	6
0.1	>95	12
0.1	>99	18

44. When using the tables the following procedure is suggested:

(a) Choose the relevant population standard.

(b) Write down the different relevant decision schemes (combinations of sample size and maximum number of off-types), with the probabilities of type I and type II errors read from the figures.

(c) Choose the decision scheme with the best balance between the probabilities of errors.

45. The use of the tables and figures is illustrated in the example section.

DETAILED DESCRIPTION OF THE METHOD FOR ONE SINGLE TEST

46. The mathematical calculations are based on the binomial distribution and it is common to use the following terms:

(a) The percentage of off-types to be accepted in a particular case is called the "population standard" and symbolized by the letter P.

(b) The "acceptance probability" is the probability of accepting a variety with P% of off-types. However, because the number of off-types is discrete, the actual probability of accepting a uniform variety varies with sample size but will always be greater than or equal to the "acceptance probability." The acceptance probability is usually denoted by 100 - α, where α is the percent probability of rejecting a variety with P% of off-types (i.e. type I error probability). In practice, many varieties will have less than P% off-types and hence the type I error will in fact be less than α for such varieties.

(c) The number of plants examined in a random sample is called the sample size and denoted by n.

(d) The maximum number of off-types tolerated in a random sample of size n is denoted by k.

(e) The probability of accepting a variety with more than P% off-types, say P_q% of off-types, is denoted by the letter β or by β_q.

(f) The mathematical formulae for calculating the probabilities are:

$$\alpha = 100 - 100 \sum_{i=0}^{k} \binom{n}{i} P^i (1-P)^{n-i} \qquad (1)$$

$$\beta_q = 100 \sum_{i=0}^{k} \binom{n}{i} P_q^i (1-P_q)^{n-i} \qquad (2)$$

P and P_q are expressed here as proportions, i.e. percents divided by 100.

MORE THAN ONE SINGLE TEST (YEAR)

47. Often a candidate variety is grown in two (or three years). The question then arises of how to combine the uniformity information from the individual years. Two methods will be described:

(a) Make the decision after two (or three) years based on the total number of plants examined and the total number of off-types recorded. (A combined test).

(b) Use the result of the first year to see if the data suggests a clear decision (reject or accept). If the decision is not clear then proceed with the second year and decide after the second year. (A two-stage test).

48. However, there are some alternatives (e.g. a decision may be made in each year and a final decision may be reached by rejecting the candidate variety if it shows too many off-types in both (or two out of three years)). Also there are complications when more than one single year test is done. It is therefore suggested that a statistician should be consulted when two (or more) year tests have to be used.

DETAILED DESCRIPTION OF THE METHODS FOR MORE THAN ONE SINGLE TEST

Combined Test

49. The sample size in test i is n_i. So after the last test we have the total sample size $n = \Sigma n_i$. A decision scheme is set in exactly the same way as if this total sample size had been obtained in a single test. Thus, the total number of off-types recorded through the tests is compared with the maximum number of off-types allowed by the chosen decision scheme.

Two-stage Test

50. The method for a two-year test may be described as follows: In the first year take a sample of size n. Reject the candidate variety if more than r_1 off-types are recorded and accept the candidate variety if less than a_1 off-types are recorded. Otherwise, proceed to the second year and take a sample of size n (as in the first year) and reject the candidate variety if the total number of off-types recorded in the two years' test is greater than r. Otherwise, accept the candidate variety. The final risks and the expected sample size in such a procedure may be calculated as follows:

$$\alpha = P(K_1 > r_1) + P(K_1 + K_2 > r \mid K_1)$$
$$= P(K_1 > r_1) + P(K_2 > r\text{-}K_1 \mid K_1)$$

$$= \sum_{i=r_1+1}^{n} \binom{n}{i} P^i (1-P)^{n-i} + \sum_{i=\alpha_1}^{r_1} \binom{n}{i} P^i (1-P)^{n-i} \sum_{j=r-i+1}^{n} \binom{n}{i} P^j (1-P)^{n-j} \qquad (3)$$

$$\beta_q = P(K_1 < \alpha_1) + P(K_1 + K_2 \le r \mid K_1)$$
$$= P(K_1 < \alpha_1) + P(K_2 \le r\text{-}K_1 \mid K_1)$$

$$= \sum_{i=0}^{\alpha_1-1} \binom{n}{i} P_q^i (1-P_q)^{n-i} + \sum_{i=\alpha_1}^{r_1} \binom{n}{i} P_q^i (1-P_q)^{n-i} \sum_{j=0}^{r-i} \binom{n}{i} P_q^j (1-P_q)^{n-j} \qquad (4)$$

$$n_e = n \left(1 + \sum_{i=\alpha_1}^{r_1} \binom{n}{i} P^i (1-P)^{n-i} \right) \qquad (5)$$

where

P = population standard
α = probability of actual type I error for P
β_q = probability of actual type II error for q P
n_e = expected sample size
r_1, a_1 and r are decision-parameters
P_q = q times population standard = q P
K_1 and K_2 are the numbers of off-types found in years 1 and 2 respectively.

51. The decision parameters, a_1, r_1 and r, may be chosen according to the following criteria:

(a) α must be less than α_0, where α_0 is the maximum type I error, i.e. α_0 is 100 minus the required acceptance probability
(b) β_q (for q=5) should be as small as possible but not smaller than α_0
(c) if β_q (for q=5) < α_0 n_e should be as small as possible.

52. However, other strategies are available. No tables/figures are produced here as there may be several different decision schemes that satisfy a certain set of risks. It is suggested that a statistician should be consulted if a 2-stage test (or any other sequential tests) is required.

SEQUENTIAL TESTS

53. The two-stage test mentioned above is a type of sequential test where the result of the first stage determines whether the test needs to be continued for a second stage. Other types of sequential tests may also be applicable. It may be relevant to consider such tests when the practical work allows analyses of off-types to be carried out at certain stages of the examination. The decision schemes for such methods can be set up in many different ways and it is suggested that a statistician should be consulted when sequential methods are to be used.

NOTE ON TYPE I AND TYPE II ERRORS

54. Because the number of off-types is discrete, we cannot in general obtain type I-errors that are nice pre-selected figures. The scheme a of example 2 with 6 plants above showed that we could not obtain an α of 10% - our actual α became 0.6%. Increasing the sample size will result in varying α and β values. Figure 3 - as an example - shows that α gets closer to its nominal values at certain sample sizes and that this is also the sample size where β is relatively small. It is also seen that increasing the sample size for fixed acceptance probability is not always advantageous. For instance a sample size of five gives $\alpha = 10\%$ and $\beta_2 = 82\%$ whereas a sample size of six gives $\alpha = 0.6\%$ and $\beta_2 = 98\%$. It appears that the sample sizes, which give α-values in close agreement with the acceptance probability are the largest in the range of sample sizes with a specified maximum number of off-types. Thus, the smallest sample sizes in the range of sample sizes with a given maximum number of off-types should be avoided.

DEFINITION OF STATISTICAL TERMS AND SYMBOLS

55. The statistical terms and symbols used have the following definitions:

Population standard. The percentage of off-types to be accepted if all the individuals of a variety could be examined. The population standard is fixed for the crop in question and is based on experience.

Acceptance probability. The probability of accepting a uniform variety with P% of off-types. Here P is population standard. However, note that the actual probability of accepting a uniform variety will always be greater than or equal to the acceptance probability in the heading of the table and figures. The probability of accepting a uniform variety and the probability of a type I error sum to 100%. For example, if the type I error probability is 4%, then the probability of accepting a uniform variety is 100 – 4 = 96%, see e.g. figure 1 for n=50). The type I error is indicated on the graph in the figures by the sawtooth peaks between 0 and the upper limit of type I error (for instance 10 on figure 1). The decision schemes are defined so that the actual probability of accepting a uniform variety is always greater than or equal to the acceptance probability in the heading of the table.

Type I error: The error of rejecting a uniform variety.

Type II error: The error of accepting a variety that is too heterogeneous.

P Population standard

P_q The assumed true percentage of off-types in a heterogeneous variety. $P_q = q\,P$.

In the present document q is equal to 2, 5 or 10. These are only 3 examples to help the visualization of type II errors. The actual percentage of off-types in a variety may take any value. For instance we may examine different varieties which in fact may have respectively 1.6%, 3.8%, 0.2%, … of off-types.

n Sample size
k Maximum number of off-types allowed
α Probability of type I error
β Probability of type II error

TABLES AND FIGURES

Table and figure 1: **Population Standard = 5%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

	n		k
1	to	2	0
3	to	10	1
11	to	22	2
23	to	35	3
36	to	49	4
50	to	63	5
64	to	78	6
79	to	94	7
95	to	109	8
110	to	125	9
126	to	141	10
142	to	158	11
159	to	174	12
175	to	191	13
192	to	207	14
208	to	224	15
225	to	241	16
242	to	258	17
259	to	275	18
276	to	292	19
293	to	310	20
311	to	327	21
328	to	344	22
345	to	362	23
363	to	379	24
380	to	397	25
398	to	414	26
415	to	432	27
433	to	449	28
450	to	467	29
468	to	485	30
486	to	503	31
504	to	520	32
521	to	538	33
539	to	556	34
557	to	574	35
575	to	592	36
593	to	610	37
611	to	628	38
629	to	646	39
647	to	664	40
665	to	682	41
683	to	700	42
701	to	718	43
719	to	736	44
737	to	754	45
755	to	772	46
773	to	791	47
792	to	809	48
810	to	827	49
828	to	845	50
846	to	864	51
865	to	882	52
883	to	900	53
901	to	918	54
919	to	937	55
938	to	955	56
956	to	973	57
974	to	992	58
993	to	1010	59

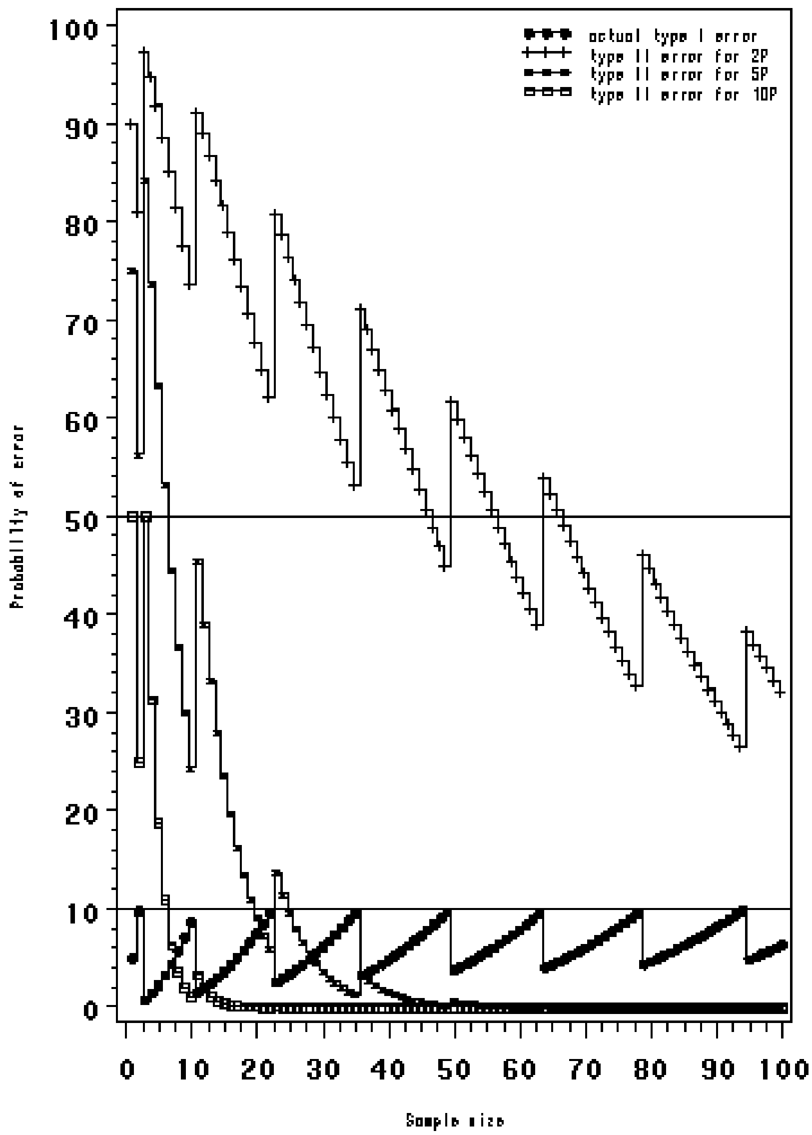


Table and figure 2: **Population Standard = 3%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n			k
1	to	3	0
4	to	17	1
18	to	37	2
38	to	58	3
59	to	81	4
82	to	105	5
106	to	130	6
131	to	156	7
157	to	182	8
183	to	208	9
209	to	235	10
236	to	262	11
263	to	289	12
290	to	317	13
318	to	345	14
346	to	373	15
374	to	401	16
402	to	429	17
430	to	457	18
458	to	486	19
487	to	515	20
516	to	543	21
544	to	572	22
573	to	601	23
602	to	630	24
631	to	659	25
660	to	689	26
690	to	718	27
719	to	747	28
748	to	777	29
778	to	806	30
807	to	836	31
837	to	865	32
866	to	895	33
896	to	925	34
926	to	955	35
956	to	984	36
985	to	1014	37
1015	to	1044	38
1045	to	1074	39
1075	to	1104	40
1105	to	1134	41
1135	to	1164	42
1165	to	1195	43
1196	to	1225	44
1226	to	1255	45
1256	to	1285	46
1286	to	1315	47
1316	to	1346	48
1347	to	1376	49
1377	to	1406	50
1407	to	1437	51
1438	to	1467	52
1468	to	1498	53
1499	to	1528	54

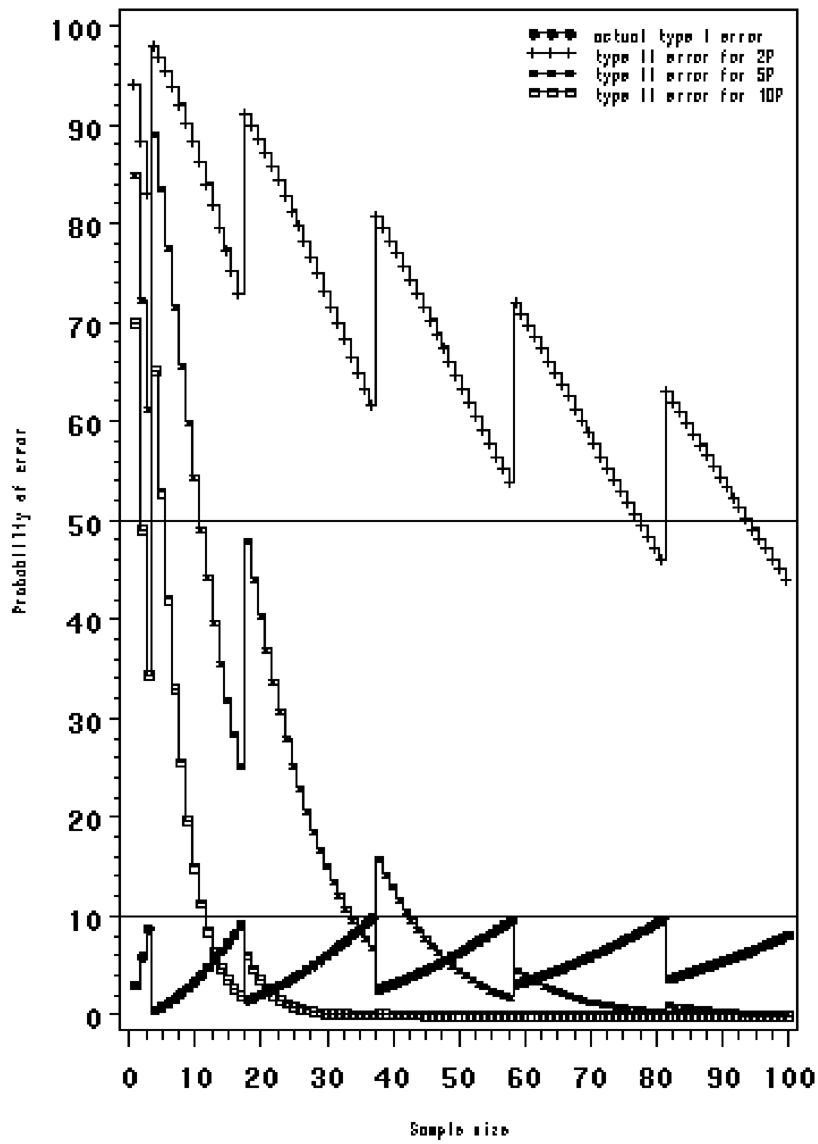


Table and figure 3: **Population Standard = 2%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n			k
1	to	5	0
6	to	26	1
27	to	55	2
56	to	87	3
88	to	122	4
123	to	158	5
159	to	195	6
196	to	233	7
234	to	272	8
273	to	312	9
313	to	352	10
353	to	393	11
394	to	433	12
434	to	475	13
476	to	516	14
517	to	558	15
559	to	600	16
601	to	643	17
644	to	685	18
686	to	728	19
729	to	771	20
772	to	814	21
815	to	857	22
858	to	901	23
902	to	944	24
945	to	988	25
989	to	1032	26
1033	to	1076	27
1077	to	1120	28
1121	to	1164	29
1165	to	1208	30
1209	to	1252	31
1253	to	1297	32
1298	to	1341	33
1342	to	1386	34
1387	to	1431	35
1432	to	1475	36
1476	to	1520	37
1521	to	1565	38
1566	to	1610	39
1611	to	1655	40
1656	to	1700	41
1701	to	1745	42
1746	to	1790	43
1791	to	1835	44
1836	to	1881	45
1882	to	1926	46
1927	to	1971	47
1972	to	2000	48

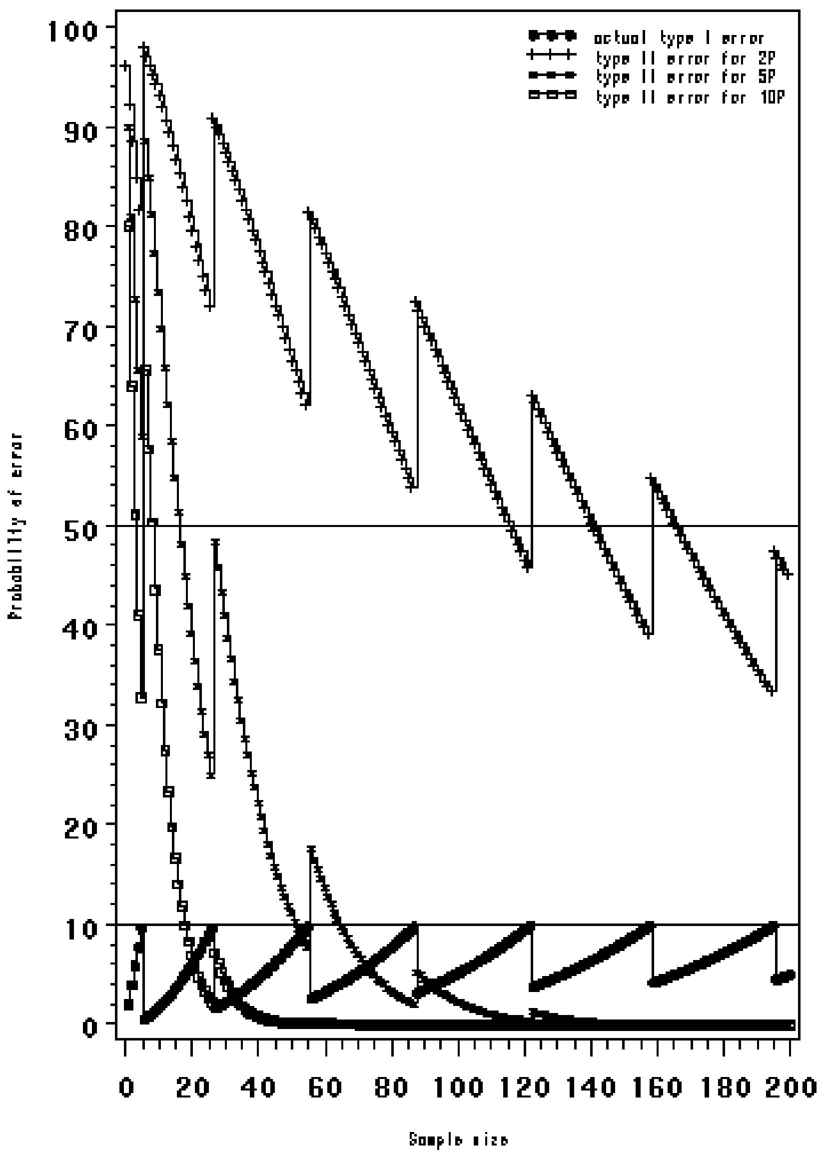


Table and figure 4: **Population Standard = 1%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n			k
1	to	10	0
11	to	53	1
54	to	110	2
111	to	175	3
176	to	244	4
245	to	316	5
317	to	390	6
391	to	466	7
467	to	544	8
545	to	623	9
624	to	703	10
704	to	784	11
785	to	866	12
867	to	948	13
949	to	1031	14
1032	to	1115	15
1116	to	1199	16
1200	to	1284	17
1285	to	1369	18
1370	to	1454	19
1455	to	1540	20
1541	to	1626	21
1627	to	1713	22
1714	to	1799	23
1800	to	1887	24
1888	to	1974	25
1975	to	2061	26
2062	to	2149	27
2150	to	2237	28
2238	to	2325	29
2326	to	2414	30
2415	to	2502	31
2503	to	2591	32
2592	to	2680	33
2681	to	2769	34
2770	to	2858	35
2859	to	2948	36
2949	to	3000	37

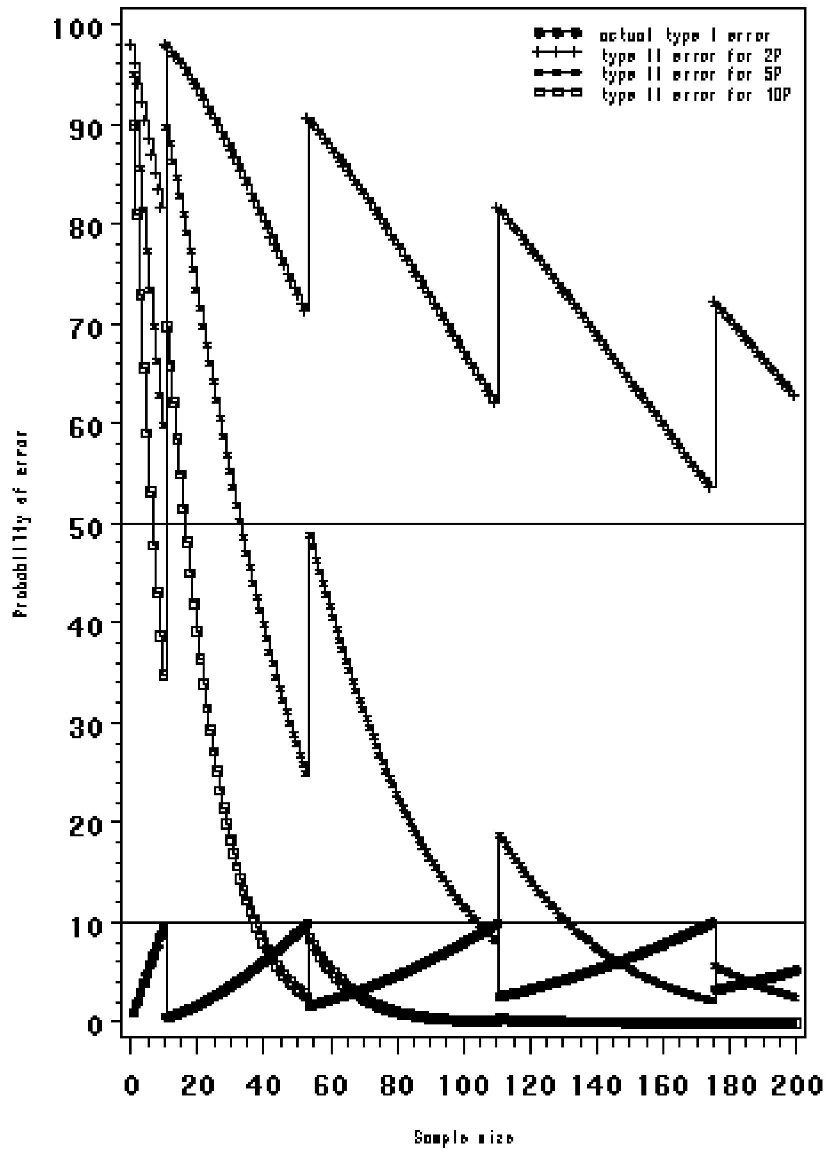


Table and figure 5: **Population Standard = .5%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n			k
1	to	21	0
22	to	106	1
107	to	220	2
221	to	349	3
350	to	487	4
488	to	631	5
632	to	780	6
781	to	932	7
933	to	1087	8
1088	to	1245	9
1246	to	1405	10
1406	to	1567	11
1568	to	1730	12
1731	to	1895	13
1896	to	2061	14
2062	to	2228	15
2229	to	2397	16
2398	to	2566	17
2567	to	2736	18
2737	to	2907	19
2908	to	3000	20

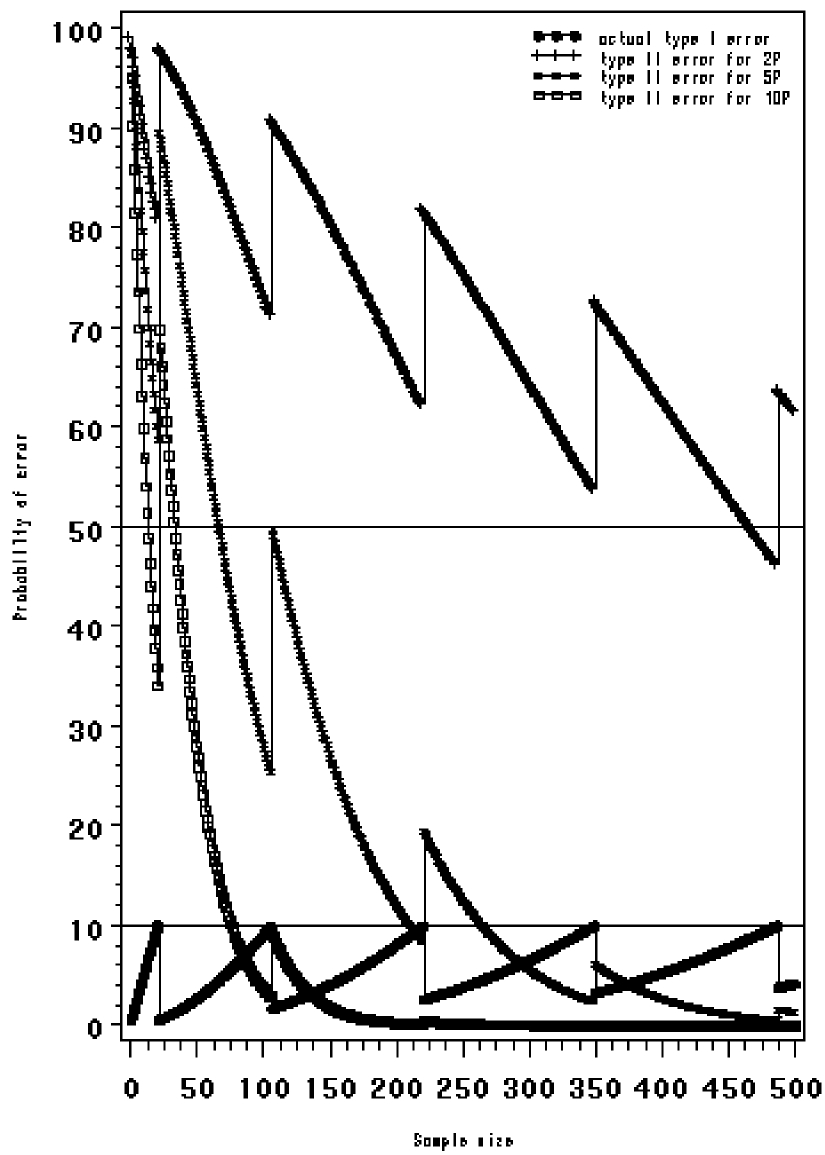


Table and figure 6: **Population Standard = .1%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

	n		k
1	to	105	0
106	to	532	1
533	to	1102	2
1103	to	1745	3
1746	to	2433	4
2434	to	3000	5

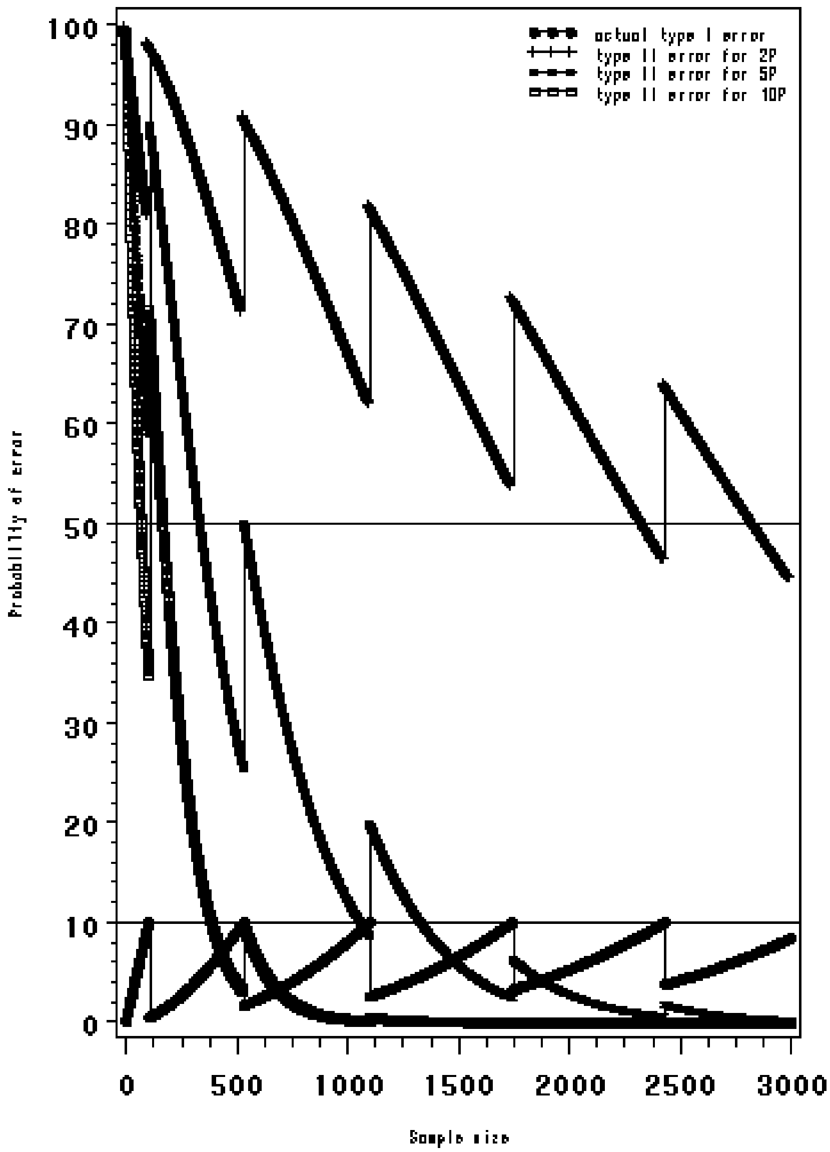


Table and figure 7: **Population Standard = 5%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n			k
1	to	1	0
2	to	7	1
8	to	16	2
17	to	28	3
29	to	40	4
41	to	53	5
54	to	67	6
68	to	81	7
82	to	95	8
96	to	110	9
111	to	125	10
126	to	140	11
141	to	155	12
156	to	171	13
172	to	187	14
188	to	203	15
204	to	219	16
220	to	235	17
236	to	251	18
252	to	268	19
269	to	284	20
285	to	300	21
301	to	317	22
318	to	334	23
335	to	351	24
352	to	367	25
368	to	384	26
385	to	401	27
402	to	418	28
419	to	435	29
436	to	452	30
453	to	469	31
470	to	487	32
488	to	504	33
505	to	521	34
522	to	538	35
539	to	556	36
557	to	573	37
574	to	590	38
591	to	608	39
609	to	625	40
626	to	643	41
644	to	660	42
661	to	678	43
679	to	696	44
697	to	713	45
714	to	731	46
732	to	748	47
749	to	766	48
767	to	784	49
785	to	802	50
803	to	819	51
820	to	837	52
838	to	855	53
856	to	873	54
874	to	891	55
892	to	909	56
910	to	926	57
927	to	944	58
945	to	962	59
963	to	980	60
981	to	998	61

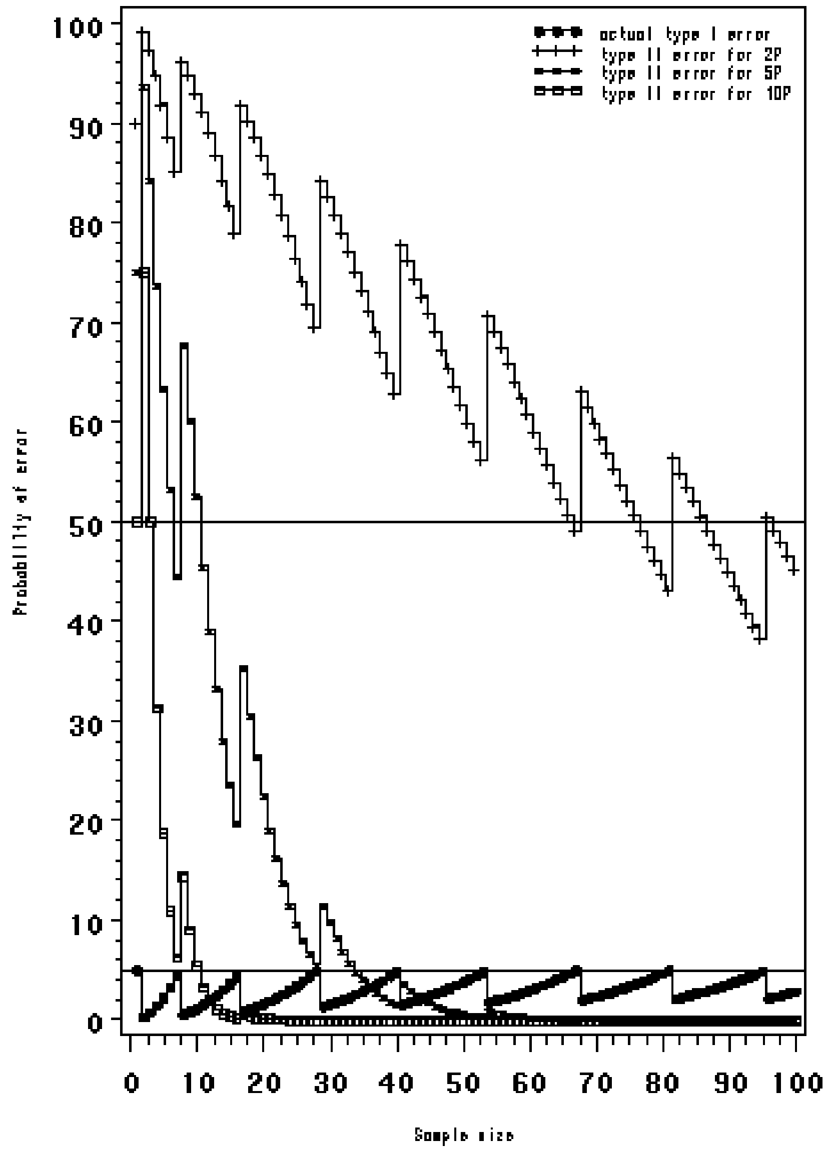


Table and figure 8: **Population Standard = 3%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

	n		k
1	to	1	0
2	to	12	1
13	to	27	2
28	to	46	3
47	to	66	4
67	to	88	5
89	to	110	6
111	to	134	7
135	to	158	8
159	to	182	9
183	to	207	10
208	to	232	11
233	to	258	12
259	to	284	13
285	to	310	14
311	to	337	15
338	to	363	16
364	to	390	17
391	to	417	18
418	to	444	19
445	to	472	20
473	to	499	21
500	to	527	22
528	to	554	23
555	to	582	24
583	to	610	25
611	to	638	26
639	to	666	27
667	to	695	28
696	to	723	29
724	to	751	30
752	to	780	31
781	to	809	32
810	to	837	33
838	to	866	34
867	to	895	35
896	to	924	36
925	to	952	37
953	to	981	38
982	to	1010	39
1011	to	1040	40
1041	to	1069	41
1070	to	1098	42
1099	to	1127	43
1128	to	1156	44
1157	to	1186	45
1187	to	1215	46
1216	to	1244	47
1245	to	1274	48
1275	to	1303	49
1304	to	1333	50
1334	to	1362	51
1363	to	1392	52
1393	to	1422	53
1423	to	1451	54
1452	to	1481	55
1482	to	1511	56
1512	to	1541	57
1542	to	1570	58
1571	to	1600	59
1601	to	1630	60
1631	to	1660	61
1661	to	1690	62
1691	to	1720	63
1721	to	1750	64
1751	to	1780	65
1781	to	1810	66
1811	to	1840	67
1841	to	1870	68
1871	to	1900	69

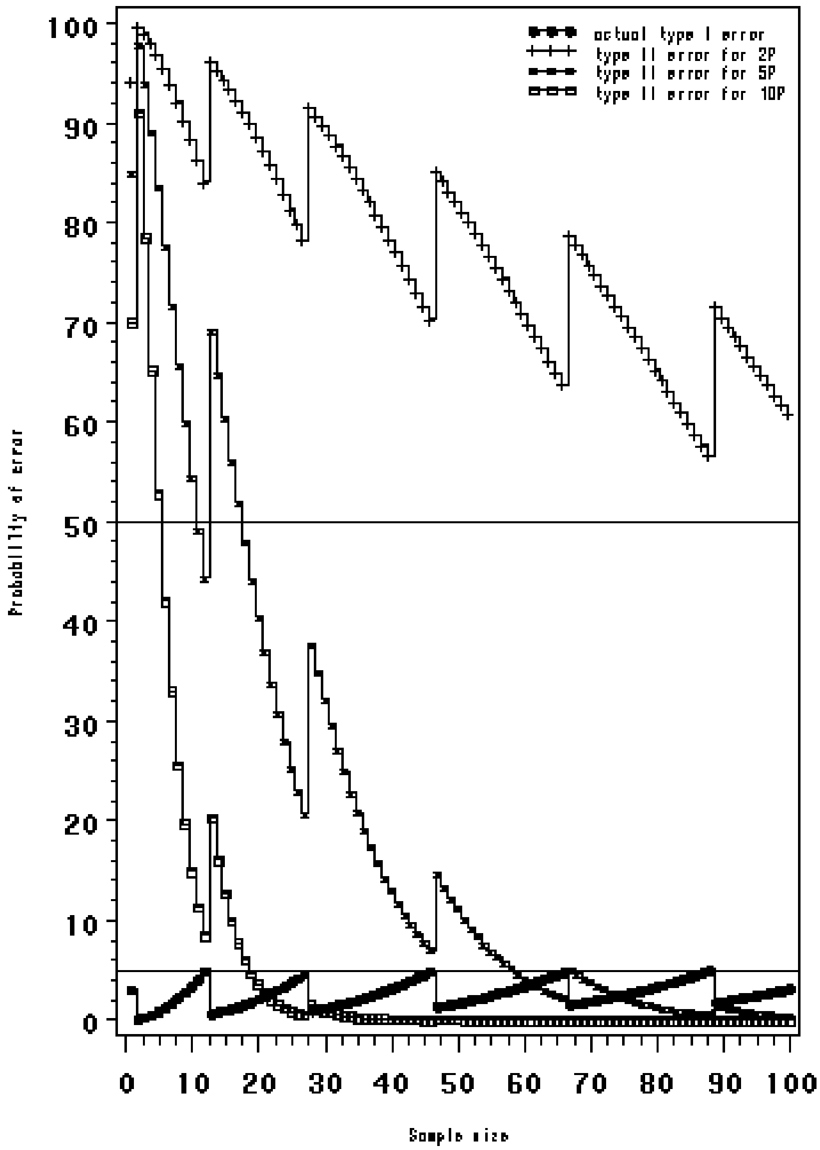


Table and figure 9: Population Standard = 2%
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n			k
1	to	2	0
3	to	18	1
19	to	41	2
42	to	69	3
70	to	99	4
100	to	131	5
132	to	165	6
166	to	200	7
201	to	236	8
237	to	273	9
274	to	310	10
311	to	348	11
349	to	386	12
387	to	425	13
426	to	464	14
465	to	504	15
505	to	544	16
545	to	584	17
585	to	624	18
625	to	665	19
666	to	706	20
707	to	747	21
748	to	789	22
790	to	830	23
831	to	872	24
873	to	914	25
915	to	956	26
957	to	998	27
999	to	1040	28
1041	to	1083	29
1084	to	1126	30
1127	to	1168	31
1169	to	1211	32
1212	to	1254	33
1255	to	1297	34
1298	to	1340	35
1341	to	1383	36
1384	to	1427	37
1428	to	1470	38
1471	to	1514	39
1515	to	1557	40
1558	to	1601	41
1602	to	1645	42
1646	to	1689	43
1690	to	1732	44
1733	to	1776	45
1777	to	1820	46
1821	to	1864	47
1865	to	1909	48
1910	to	1953	49
1954	to	1997	50
1998	to	2000	51

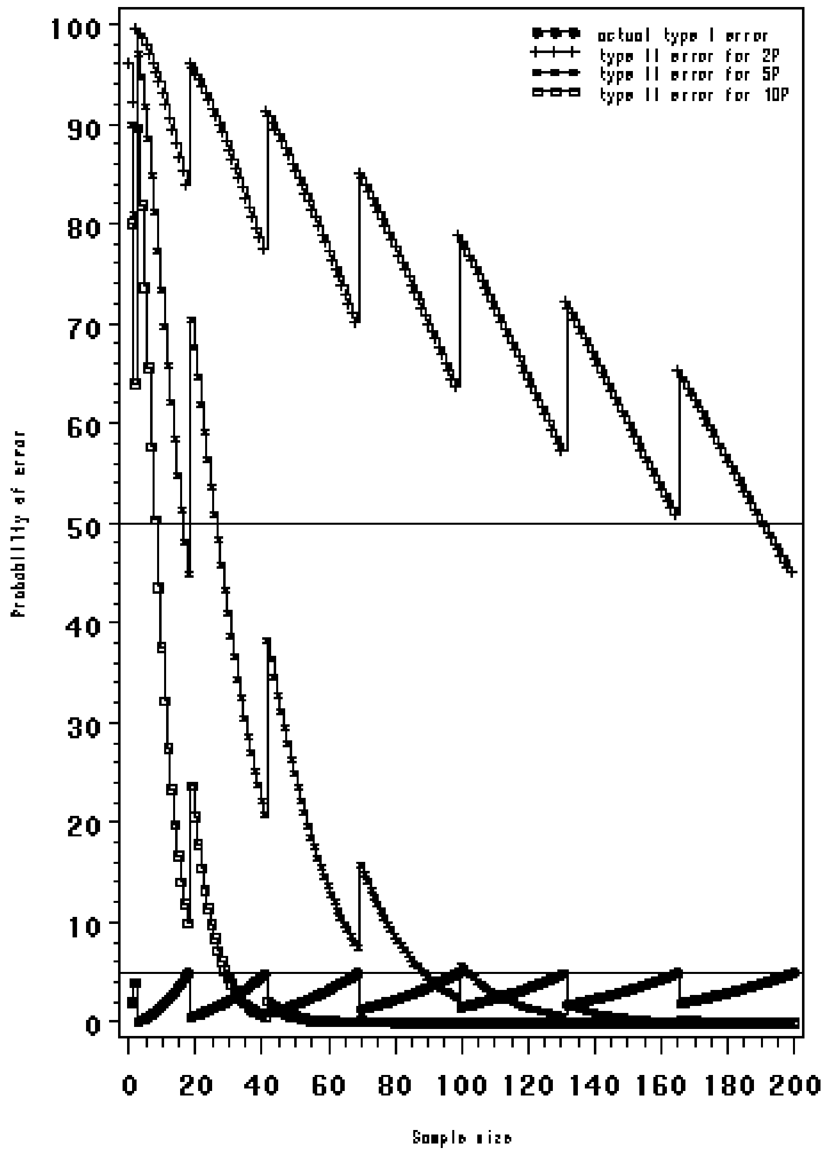


Table and figure 10: **Population Standard = 1%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n			k
1	to	5	0
6	to	35	1
36	to	82	2
83	to	137	3
138	to	198	4
199	to	262	5
263	to	329	6
330	to	399	7
400	to	471	8
472	to	544	9
545	to	618	10
619	to	694	11
695	to	771	12
772	to	848	13
849	to	927	14
928	to	1006	15
1007	to	1085	16
1086	to	1166	17
1167	to	1246	18
1247	to	1328	19
1329	to	1410	20
1411	to	1492	21
1493	to	1575	22
1576	to	1658	23
1659	to	1741	24
1742	to	1825	25
1826	to	1909	26
1910	to	1993	27
1994	to	2078	28
2079	to	2163	29
2164	to	2248	30
2249	to	2333	31
2334	to	2419	32
2420	to	2505	33
2506	to	2591	34
2592	to	2677	35
2678	to	2763	36
2764	to	2850	37
2851	to	2937	38
2938	to	3000	39

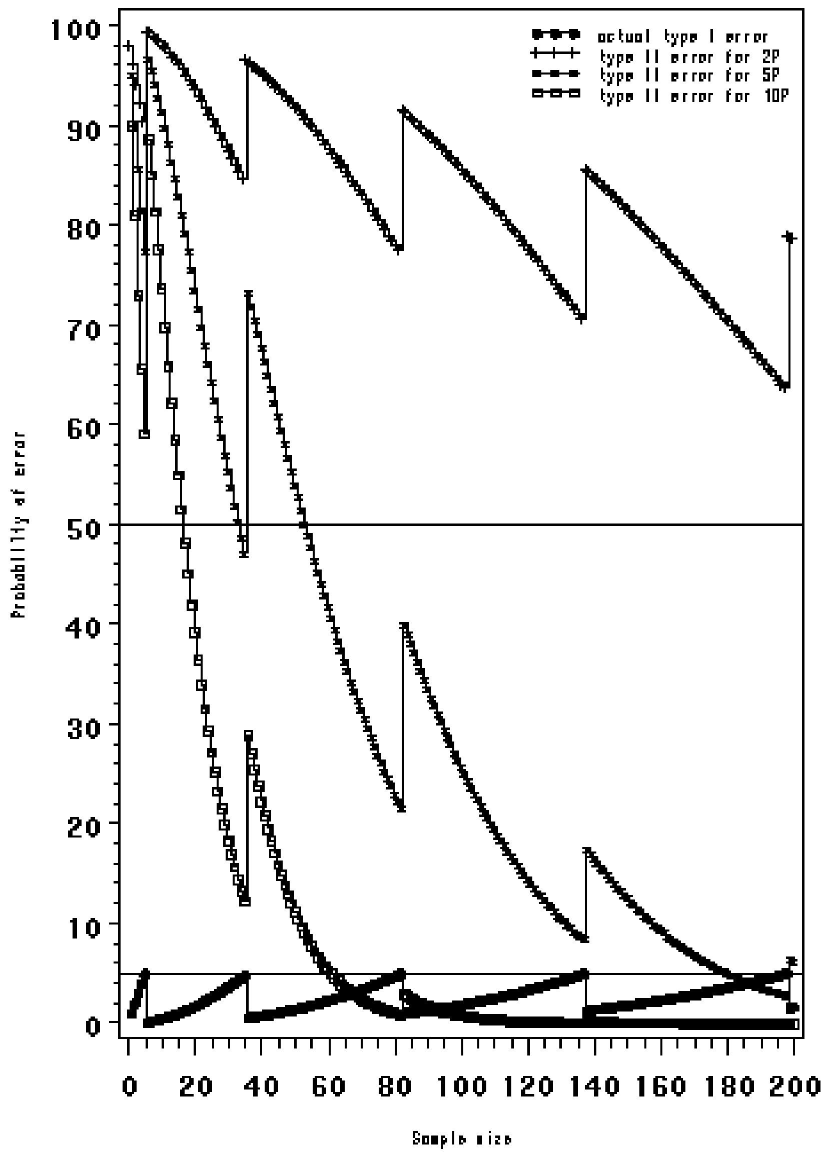


Table and figure 11: **Population Standard = .5%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n			k
1	to	10	0
11	to	71	1
72	to	164	2
165	to	274	3
275	to	395	4
396	to	523	5
524	to	658	6
659	to	797	7
798	to	940	8
941	to	1086	9
1087	to	1235	10
1236	to	1386	11
1387	to	1540	12
1541	to	1695	13
1696	to	1851	14
1852	to	2009	15
2010	to	2169	16
2170	to	2329	17
2330	to	2491	18
2492	to	2653	19
2654	to	2817	20
2818	to	2981	21
2982	to	3000	22

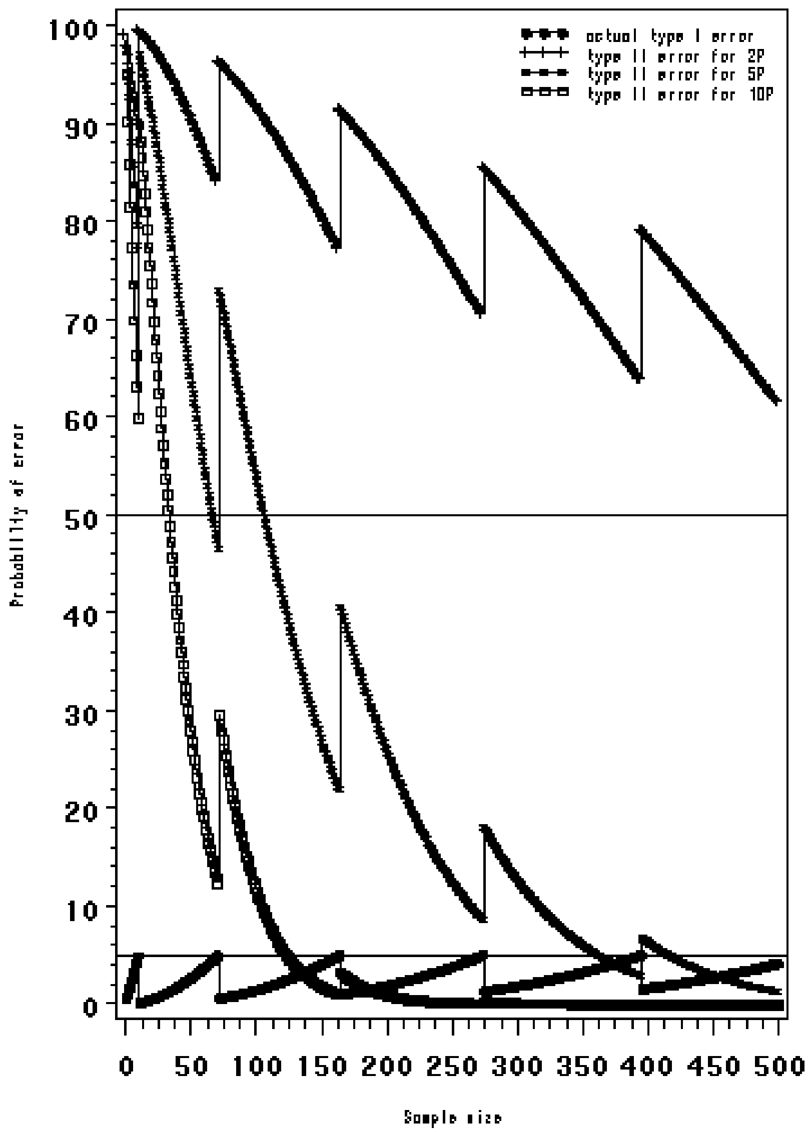


Table and figure 12: **Population Standard = .1%**
Acceptance Probability ≥95%
n=sample size, k=maximum number off-types

n			k
1	to	51	0
52	to	355	1
356	to	818	2
819	to	1367	3
1368	to	1971	4
1972	to	2614	5
2615	to	3000	6

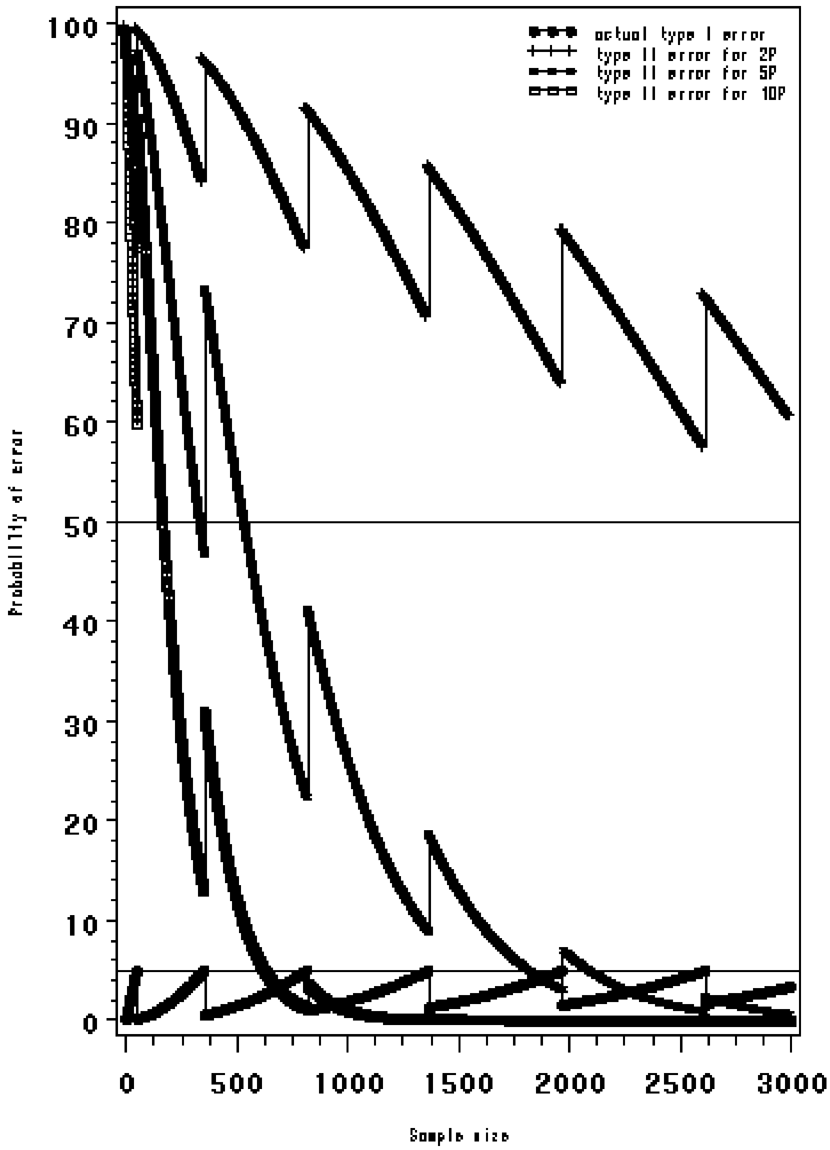


Table and figure 13: **Population Standard = 5%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

n			k
1	to	3	1
4	to	9	2
10	to	17	3
18	to	26	4
27	to	37	5
38	to	48	6
49	to	60	7
61	to	72	8
73	to	85	9
86	to	98	10
99	to	111	11
112	to	124	12
125	to	138	13
139	to	152	14
153	to	167	15
168	to	181	16
182	to	196	17
197	to	210	18
211	to	225	19
226	to	240	20
241	to	255	21
256	to	270	22
271	to	286	23
287	to	301	24
302	to	317	25
318	to	332	26
333	to	348	27
349	to	364	28
365	to	380	29
381	to	395	30
396	to	411	31
412	to	427	32
428	to	444	33
445	to	460	34
461	to	476	35
477	to	492	36
493	to	508	37
509	to	525	38
526	to	541	39
542	to	558	40
559	to	574	41
575	to	591	42
592	to	607	43
608	to	624	44
625	to	640	45
641	to	657	46
658	to	674	47
675	to	690	48
691	to	707	49
708	to	724	50
725	to	741	51
742	to	758	52
759	to	775	53
776	to	792	54
793	to	809	55
810	to	826	56
827	to	843	57
844	to	860	58
861	to	877	59
878	to	894	60
895	to	911	61
912	to	928	62
929	to	945	63
946	to	962	64
963	to	979	65
980	to	997	66
998	to	1014	67

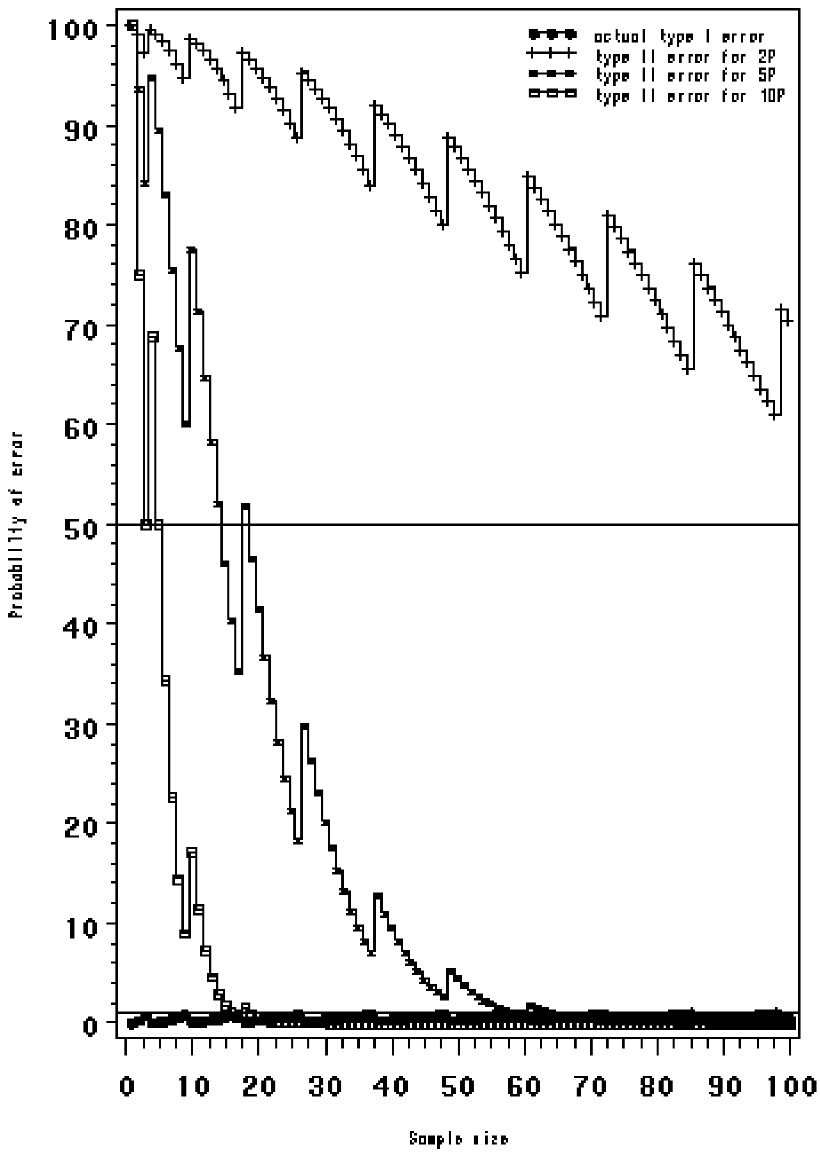


Table and figure 14: **Population Standard = 3%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

	n		k
1	to	5	1
6	to	15	2
16	to	28	3
29	to	44	4
45	to	61	5
62	to	79	6
80	to	98	7
99	to	119	8
120	to	140	9
141	to	161	10
162	to	183	11
184	to	206	12
207	to	229	13
230	to	252	14
253	to	276	15
277	to	300	16
301	to	324	17
325	to	348	18
349	to	373	19
374	to	398	20
399	to	423	21
424	to	448	22
449	to	474	23
475	to	499	24
500	to	525	25
526	to	551	26
552	to	577	27
578	to	603	28
604	to	629	29
630	to	656	30
657	to	682	31
683	to	709	32
710	to	736	33
737	to	763	34
764	to	789	35
790	to	816	36
817	to	844	37
845	to	871	38
872	to	898	39
899	to	925	40
926	to	953	41
954	to	980	42
981	to	1008	43
1009	to	1035	44
1036	to	1063	45
1064	to	1091	46
1092	to	1119	47
1120	to	1146	48
1147	to	1174	49
1175	to	1202	50
1203	to	1230	51
1231	to	1258	52
1259	to	1286	53
1287	to	1315	54
1316	to	1343	55
1344	to	1371	56
1372	to	1399	57
1400	to	1428	58
1429	to	1456	59
1457	to	1484	60
1485	to	1513	61

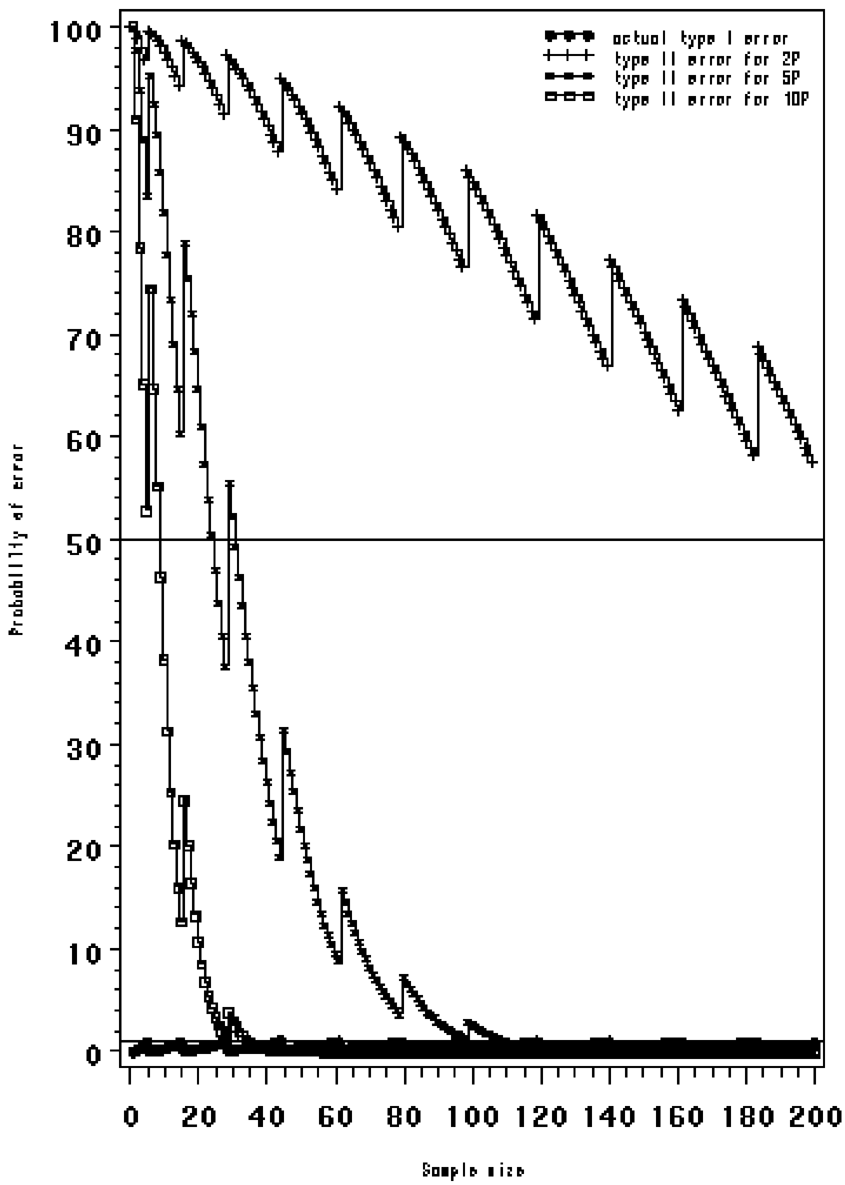


Table and figure 15: **Population Standard = 2%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

	n		k
1	to	7	1
8	to	22	2
23	to	42	3
43	to	65	4
66	to	90	5
91	to	118	6
119	to	147	7
148	to	177	8
178	to	208	9
209	to	241	10
242	to	274	11
275	to	307	12
308	to	342	13
343	to	377	14
378	to	412	15
413	to	448	16
449	to	484	17
485	to	521	18
522	to	558	19
559	to	595	20
596	to	632	21
633	to	670	22
671	to	708	23
709	to	747	24
748	to	785	25
786	to	824	26
825	to	863	27
864	to	902	28
903	to	942	29
943	to	981	30
982	to	1021	31
1022	to	1061	32
1062	to	1101	33
1102	to	1141	34
1142	to	1182	35
1183	to	1222	36
1223	to	1263	37
1264	to	1303	38
1304	to	1344	39
1345	to	1385	40
1386	to	1426	41
1427	to	1467	42
1468	to	1509	43
1510	to	1550	44
1551	to	1591	45
1592	to	1633	46
1634	to	1675	47
1676	to	1716	48
1717	to	1758	49
1759	to	1800	50
1801	to	1842	51
1843	to	1884	52
1885	to	1926	53
1927	to	1968	54
1969	to	2000	55

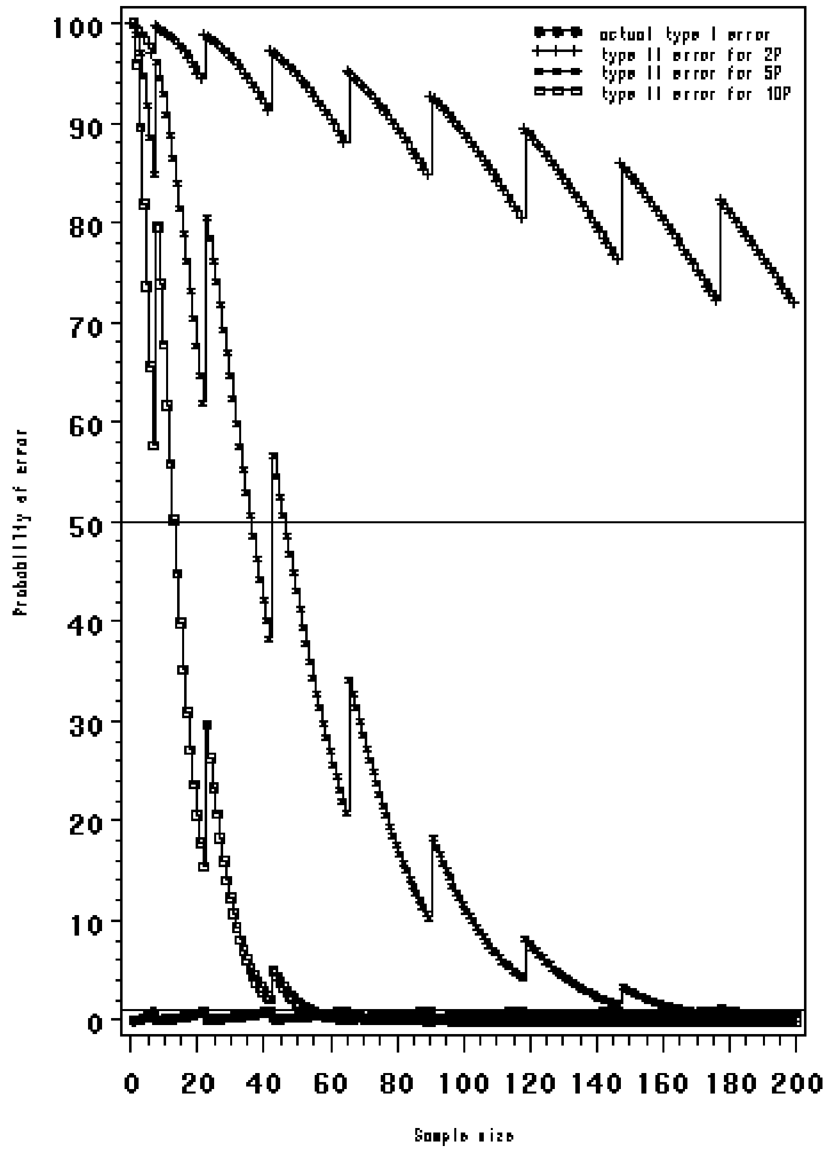


Table and figure 16: **Population Standard = 1%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

	n		k
1	to	1	0
2	to	15	1
16	to	44	2
45	to	83	3
84	to	129	4
130	to	180	5
181	to	234	6
235	to	292	7
293	to	353	8
354	to	415	9
416	to	479	10
480	to	545	11
546	to	612	12
613	to	681	13
682	to	750	14
751	to	821	15
822	to	893	16
894	to	965	17
966	to	1038	18
1039	to	1112	19
1113	to	1186	20
1187	to	1261	21
1262	to	1337	22
1338	to	1413	23
1414	to	1489	24
1490	to	1566	25
1567	to	1644	26
1645	to	1722	27
1723	to	1800	28
1801	to	1879	29
1880	to	1958	30
1959	to	2037	31
2038	to	2117	32
2118	to	2197	33
2198	to	2277	34
2278	to	2358	35
2359	to	2439	36
2440	to	2520	37
2521	to	2601	38
2602	to	2683	39
2684	to	2764	40
2765	to	2846	41
2847	to	2929	42
2930	to	3000	43

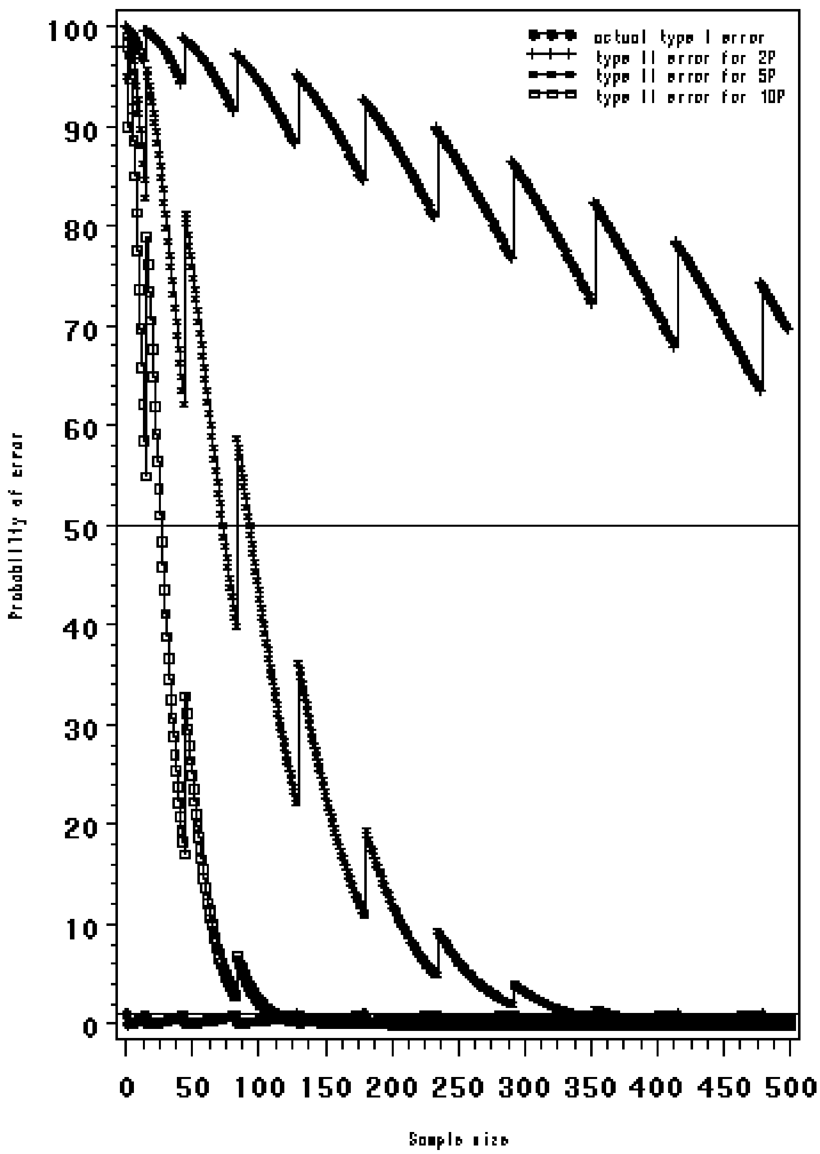


Table and figure 17: **Population Standard = .5%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

n			k
1	to	2	0
3	to	30	1
31	to	87	2
88	to	165	3
166	to	257	4
258	to	358	5
359	to	467	6
468	to	583	7
584	to	703	8
704	to	828	9
829	to	956	10
957	to	1088	11
1089	to	1222	12
1223	to	1359	13
1360	to	1498	14
1499	to	1639	15
1640	to	1782	16
1783	to	1926	17
1927	to	2072	18
2073	to	2220	19
2221	to	2369	20
2370	to	2519	21
2520	to	2670	22
2671	to	2822	23
2823	to	2975	24
2976	to	3000	25

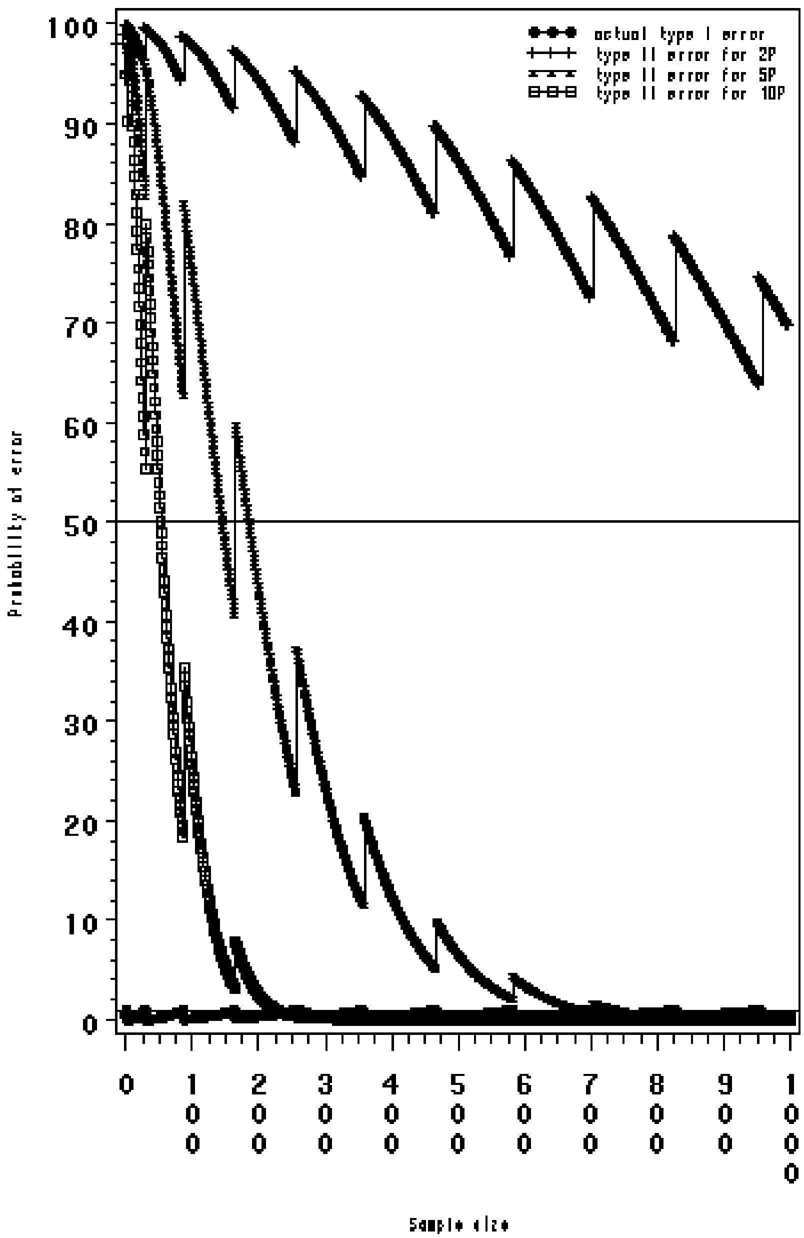


Table and figure 18: **Population Standard = .1%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

	n		k
1	to	10	0
11	to	148	1
149	to	436	2
437	to	824	3
825	to	1280	4
1281	to	1786	5
1787	to	2332	6
2333	to	2908	7
2909	to	3000	8

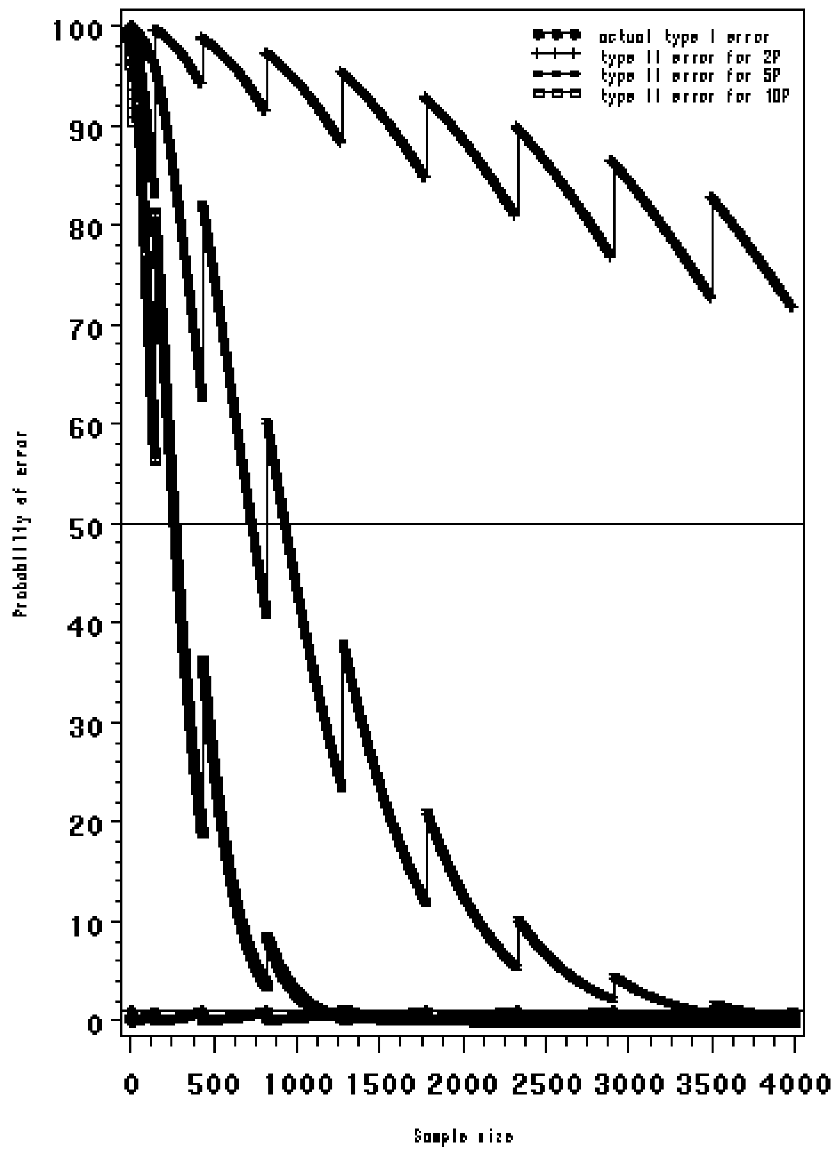


Table and figure 19: **Population Standard = 10%**
Acceptance Probability ≥ 90%
n=sample size, k=maximum number of off-types

n			k
1	to	1	0
2	to	5	1
6	to	11	2
12	to	18	3
19	to	25	4
26	to	32	5
33	to	40	6
41	to	47	7
48	to	55	8
56	to	63	9
64	to	71	10
72	to	79	11
80	to	88	12
89	to	96	13
97	to	104	14
105	to	113	15
114	to	121	16
122	to	130	17
131	to	138	18
139	to	147	19
148	to	156	20
157	to	164	21
165	to	173	22
174	to	182	23
183	to	191	24
192	to	199	25
200	to	200	26

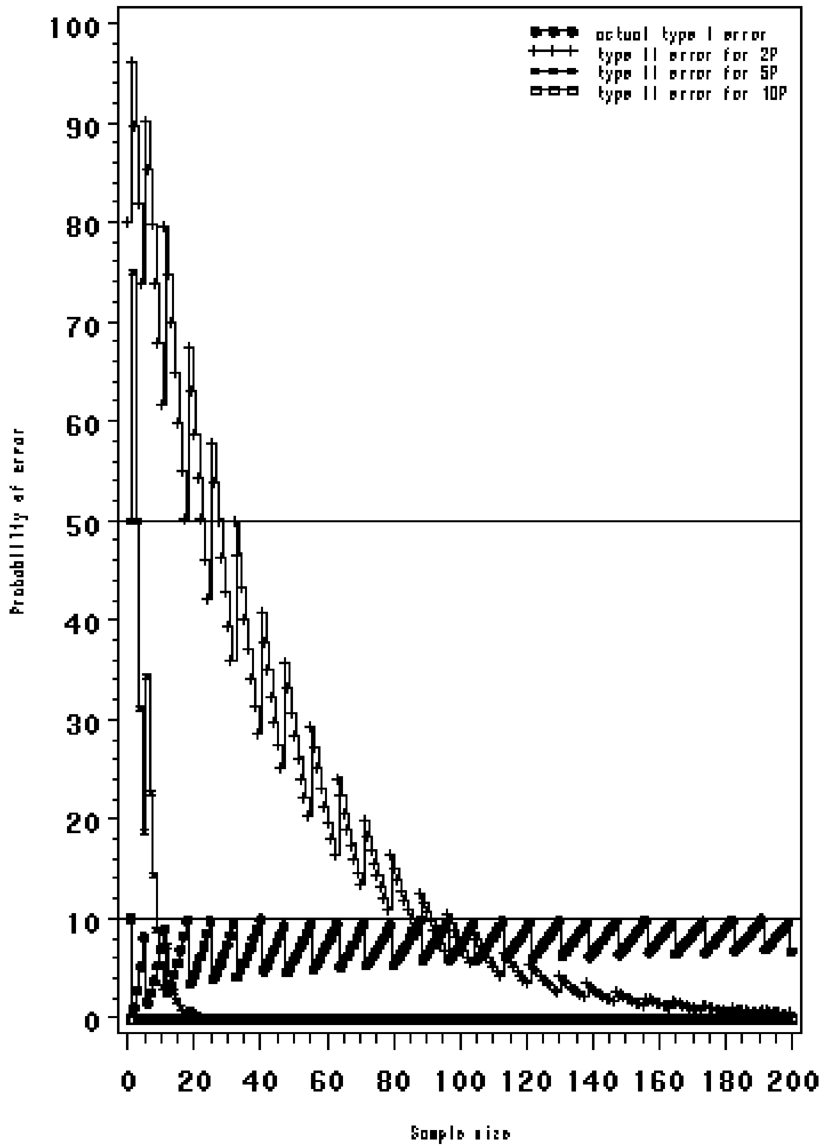


Table and figure 20: **Population Standard = 10%**
Acceptance Probability ≥ 95%
n=sample size, k=maximum number of off-types

	n		k
1	to	3	1
4	to	8	2
9	to	14	3
15	to	20	4
21	to	27	5
28	to	34	6
35	to	41	7
42	to	48	8
49	to	56	9
57	to	63	10
64	to	71	11
72	to	79	12
80	to	86	13
87	to	94	14
95	to	102	15
103	to	110	16
111	to	119	17
120	to	127	18
128	to	135	19
136	to	143	20
144	to	152	21
153	to	160	22
161	to	168	23
169	to	177	24
178	to	185	25
186	to	194	26
195	to	200	27

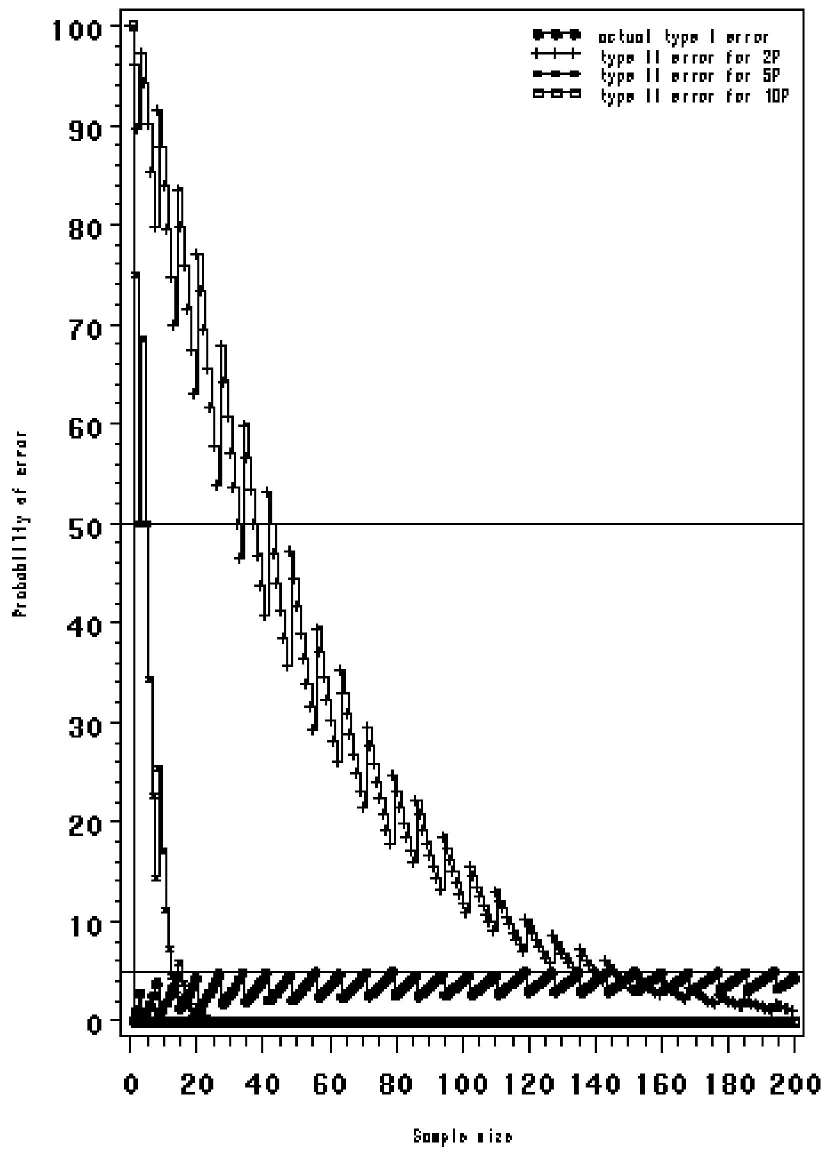
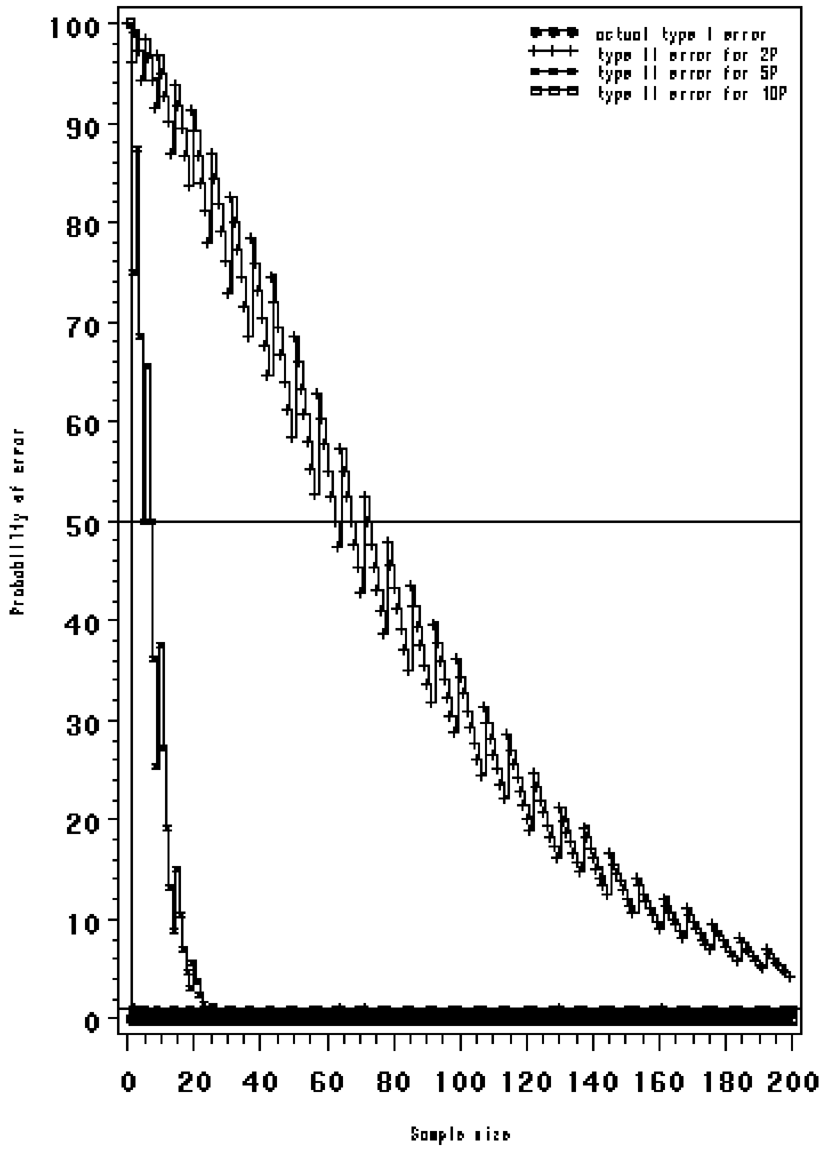


Table and figure 21 : **Population Standard = 10%**
Acceptance Probability ≥ 99%
n=sample size, k=maximum number of off-types

	n		k
1	to	2	1
3	to	5	2
6	to	9	3
10	to	14	4
15	to	19	5
20	to	25	6
26	to	31	7
32	to	37	8
38	to	43	9
44	to	50	10
51	to	57	11
58	to	64	12
65	to	71	13
72	to	78	14
79	to	85	15
86	to	92	16
93	to	99	17
100	to	107	18
108	to	114	19
115	to	122	20
123	to	130	21
131	to	137	22
138	to	145	23
146	to	153	24
154	to	161	25
162	to	168	26
169	to	176	27
177	to	184	28
185	to	192	29
193	to	200	30



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